THE PROBABILITY SCENARIO-BASED APPROACH FOR A VEHICLE ROUTING PROBLEM WITH UNCERTAIN TRAVEL TIMES

วิธีการสร้างสถานการณ์แบบทราบโอกาสในการเกิดสำหรับปัญหาการจัดเส้นทางการเดินรถขนส่งด้วยเวลาเดินทางที่ไม่แน่นอน

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ABSTRACT

The main objective of this paper is to find a robust solution of a vehicle routing problem (VRP) with uncertain travel times that minimizes the relative ratio, *R*. The ratio *R* is a performance measure. It quantifies the relative extra travel times of the robust solution with respect to the optimum travel times for each realization scenarios. This paper presents a new approach, the probability scenario-based approach (PSBA). The developed VRP model reflects the intrinsic difficulties to estimate travel times exactly in reality such as traffic conditions, accidents, traffic jams or weather conditions. The critical element of this paper is model which incorporate uncertainty and so the robustness solution approach for handling uncertainty. The experiment sets a VRP problem with the percentage of uncertainty, β , while $0 \leq \beta \leq 1$. The assumption is that the VRP model can be estimated the travel times by using 10 scenarios with known probability. The performance of the approach is compared with the well-known approaches: the scenario-based approach (SBA) and the worst-case approach (WCA). The results clearly show that the PSBA performs well against perturbation of traffic conditions.

KEYWORDS : Vehicle routing problem, Uncertain travel times, Probability scenario-based approach, Robustness, Memetic algorithm

บทคัดย่อ

วัตถุประสงค์หลักของงานวิจัยนี้ คือ การหาผลเฉลยเชิงทนทานสำหรับปัญหาการจัดเส้นทางเดินรถขนส่งที่มีเวลาเดินทางไม่แน่นอน ค่าเป้าหมาย คือ สัดส่วนสัมพัทธ์ *R* ต่ำสุด โดยที่สัดส่วนสัมพัทธ์ *R* คือ ดัชนีบ่งชี้สมรรถนะของวิธีการเข้าสู่ผลเฉลยซึ่งวัดจากเวลาเดินทางสัมพัทธ์ระหว่างผลเฉลยเชิงทนทานกับผลเฉลยเหมาะสุดเมื่อสถานการณ์นั้นๆ เกิดขึ้นจริง งานวิจัยนี้ได้นำเสนอวิธีการเข้าสู่ผลเฉลยแบบใหม่ เรียกว่า วิธีการสร้างสถานการณ์แบบทราบโอกาสในการเกิด สำหรับรูปแบบปัญหาการจัดเส้นทางในกรณีนี้ในความเป็นจริงไม่สามารถหาเวลาเดินทางที่แน่นอนได้ อันเนื่องมาจากสภาพการจราจรที่คับคั่ง การเกิดอุบัติเหตุ หรือสภาวะอากาศเลวร้าย เป็นต้น ประเด็นสำคัญของงานวิจัยนี้ คือ การสร้างรูปแบบของความไม่แน่นอน โดยใช้หลักการเข้าสู่ผลเฉลยเชิงทนทาน ซึ่งผลเฉลยที่ได้จะคงทนต่อความไม่แน่นอน การทดลองได้กำหนดปัญหาการจัดเส้นทางเดินรถขนส่ง ด้วยเปอร์เซ็นต์ความไม่แน่นอนของสภาพการจราจร, β เมื่อ $0 \leq \beta \leq 1$ โดยสมมุติฐานในการกำหนดรูปแบบปัญหา คือ สถานการณ์การเกิดสภาพการจราจรแบบต่างๆ สามารถแบ่งได้เป็น 10 สถานการณ์ และทราบโอกาสในการเกิดเหตุการณ์ต่างๆ จากนั้นทำการเปรียบเทียบสมรรถนะของวิธีการกับวิธีที่เป็นที่รู้จัก และใช้กันอย่างแพร่หลาย ได้แก่ วิธีการสร้างสถานการณ์แบบไม่กำหนดโอกาสในการเกิด และวิธีแบบกรณีแย่ที่สุด ผลการทดลองแสดงให้เห็นว่าวิธีการสร้างสถานการณ์แบบทราบโอกาสในการเกิดมีสมรรถนะที่ดีกว่า กล่าวคือ สามารถให้ผลเฉลยที่มีความทนทานต่อความไม่แน่นอนได้ดีกว่า

คำสำคัญ : ปัญหาการจัดเส้นทางเดินรถขนส่ง เวลาเดินทางไม่แน่นอน วิธีการสร้างสถานการณ์แบบ ทราบโอกาสในการเกิด ความทนทาน ขั้นตอนวิธีการมิมิติค

Introduction

The vehicle routing problem (VRP) is defined as the problem of routing a fleet of vehicles from a depot to service a set of customers that are geographically dispersed. This type of problem becomes complex when travel times are uncertain. Therefore, there is a need to develop routing and scheduling tools that account directly for the uncertainty. Recently, researchers have begun to study such problems and developed approaches for finding robust solutions which have the best worst-case performance over a set of possible scenarios. Kouvelis and Yu (1977) discuss approaches for handling uncertainty and review robust discrete optimization problems and its applications.

This paper proposes the probability scenario-based approach (PSBA) by using a memetic algorithm to solve the VRP. Its objective is to find the robust route that minimizes the relative ratio, *R*. The traditional approaches for handling uncertainty in decision making have been divided conventionally into two categories: the stochastic optimization approach and the robust optimization approach. The stochastic optimization approach does recognize the presence of multiple data instances that might be potentially realized in the future. However, the failure of it is the best solution has been selected in the expected outcome which the exactly known probability. Otherwise, the robustness approach, one that performs well across all uncertain data, It hedges against the worst case of all possible scenarios. The weak point of the robustness approach is the overly conservative of the solution that did not occur in the real system. This paper presents the PSBA concept for VRP problem that concentrates between stochastic and robustness approaches.

Traffic congestion is one of the transportation problems in large cities. It is an important cause of uncertain travel times between one city and another. The decision makers, who are the vehicles schedule planner, try to find the robust routes for their drivers which make a skilled driver, safety and punctual delivery. The determining for a robust vehicle routing with significant traffic conditions, the decision makers have to deal with uncertain travel times. The uncertainty estimates are assigned to each arc of the network and then solved by an appropriate approach.

Experimental results show that the PSBA can provide robust solutions. Then the traffic condition is utilized to evaluate the effectiveness of the approach. The experiment simulates the instance of a travel time by generating random elements from a uniform distribution, and uses them to evaluate the solutions under the relative criteria. The performance of the PSBA is evaluated by comparing its results with the scenario-based approach (SBA) and the worst-case approach (WCA).

The rest of this paper is organized as follow: The literature review focuses on literature which is relevant to the vehicle routing problem with uncertain travel times. The problem formulation and solution method section explain how the problem is formulated and solved. The sample problem section explains the problem and method to solve this problem. The results and discussion show the results from the experiment. Finally, conclusions and future work are explained in the last section.

Literature Review

The VRP arises in retail distributions such as school bus routing, mail and newspaper delivery, municipal waste collection, fuel oil delivery, dial-a-ride service, airline and railway fleet routing and scheduling. The VRP is defined on a graph, $G=(V,A)$, where $V=\{\upsilon_1,\upsilon_2,...,\upsilon_n\}$ is a set of vertices and $A \subseteq \{(\upsilon_i,\upsilon_j): i \neq j, \upsilon_i, \upsilon_j \in V\}$ is the arcs set. An optimal set of routes is composed by cyclic linkage of arcs. The routes start and end at a depot that can serve a demand for a given set of customers at each vertex (Shen et al., 2006). It is an NP-complete problem and a hard combinatorial problem which, is usually solved by heuristic and meta-heuristic methods (Luiz *et al.,* 2004).

Therefore, VRP is more complex as it involves the uncertain parameters making it more difficult to find the optimize solution. A field of decision-making under uncertainty is pioneered in the 1950s by Dantzig (1955) along with Charnes and Cooper (1959), who set the foundation stochastic programming and optimization under probabilistic constraints. Nowadays, stochastic programming has established for solving a problem which consists of uncertainty, called random elements, efficiently (Montemanni *et al.,* 2007). The uncertainty can be presented in different parts of VRP. This research focuses on the uncertain travel times because of their existences in a large city especially Bangkok of Thailand, which has high density traffic.

Recently, the researchers are more interested in a robust optimization approach. The structuring of data uncertainty is a part of critical elements in the applications of the robust approach but its probability density function is ignored (Tharinee, 2009). Sungur (2008) presents the derivations of the RVRP formulations for problems with demand uncertainty and shows that for the Miller-Tucker-Zemlin (MTZ) formulation and demand uncertainty sets constructed from combinations of scenarios. Moghaddam and Sadjadi (2009) use this MTZ formulation of the VRP and a general uncertainty sets to model a VRP with uncertainty in demands. The computational results show that the robust solution can be protected from unmet demand while incurring a small additional cost over deterministic optimal routes. Liu and Lai (2009) consider VRP with uncertain demand. The approximate reasoning algorithm is developed to determine the preference strength to send the vehicle to the next node, and the improved sweeping algorithm with vehicle coordinated strategy is originally proposed to determine a set of vehicle routes that minimizes costs. Finally, the computational results are presented to show the high effectiveness and performance of the solution approaches. Haghani and Jung (2005) present a formulation for the dynamic vehicle routing problem with time-dependent travel times. They use a genetic algorithm to solve the problem. In their additional tests on a simulated network, the proposed algorithm works well in dealing with situations in which accidents cause significant congestion in some parts of the transportation network. Herrmann (1999) applies the scenario-based approach to model the uncertainty and uses a two-space genetic algorithm to find the optimal makespan for the robust parallel machine scheduling problem. Montemanni *et al.*, (2007) present a new extension to the traveling salesman problem (TSP), where the travel times are specified as a range of possible values. They apply the robust deviation criterion and the exact methods to solve optimization. These exact methods are available for small scale problems but they might not be feasible for the large scale problems.

The next section describes the method to solve the VRP with uncertain travel times where a travel time is defined as a finite interval.

Methods

In this paper, the PSBA and memetic algorithm based on the minimax optimization approach are used. The PSBA represents the uncertainty through a finite set of uncertainty. Its model can be seen as a snapshot representing the transportation network situation as a path of any possible edge cost configuration.

A robust discrete optimization problem can be formulated as follows. Let X be the set of all solutions. Let S be the set of all possible scenarios, and p is known as a probability of the scenario. The performance of the solution is $x \in X$, the probability, $0 < p < 1$ and the scenario, $s \in S$ is $F(x, p, s)$. The problem is to find the solution that has the best robustness performance, which is the same as minimizing (over all solutions) the maximum (over all scenarios) performance.

$$\min_{x \in X} \min_{s \in S} F(x, p, s) \tag{1}$$

The objective function (1) is the general form of the robust discrete optimization. In order to describe formally all definitions for this problem, they are explained as follows:

Definition 1: A scenario S is realization of the arc travel times, i.e. t_{ij} is the travel times for each pair of customers i to j. An interval of the travel times is defined as $[l_{ij}, u_{ij}]$ where l_{ij} is a minimum travel time and u_{ij} is maximum travel time. In a base case, scenario s is randomly generated from a $[l_{ij}, u_{ij}]$.

SBA is an approach that uses the scenario method representing t_{ij} for each arcs set. The PSBA is an extension of SBA with known probability, and WCA is the maximum travel times, $t_{ij} = u_{ij}$. A simple problem shows in Figure 1-3.

Definition 2: The relative criteria, Z_R measures a percentage deviation of the robust solution, x_R from optima among all feasible decisions over all input data scenarios with known probability, p.

$$Z_R = \min_{x \in X} \max_{s \in S} \frac{f(x_R, t_{p,s}) - f(x_s^*, t_{p,s})}{f(x_s^*, t_{p,s})} \tag{2}$$

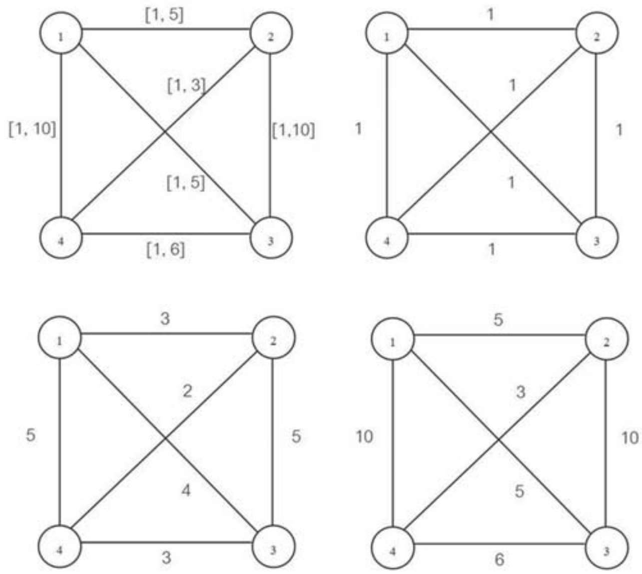

Figure 1 SBA Approach

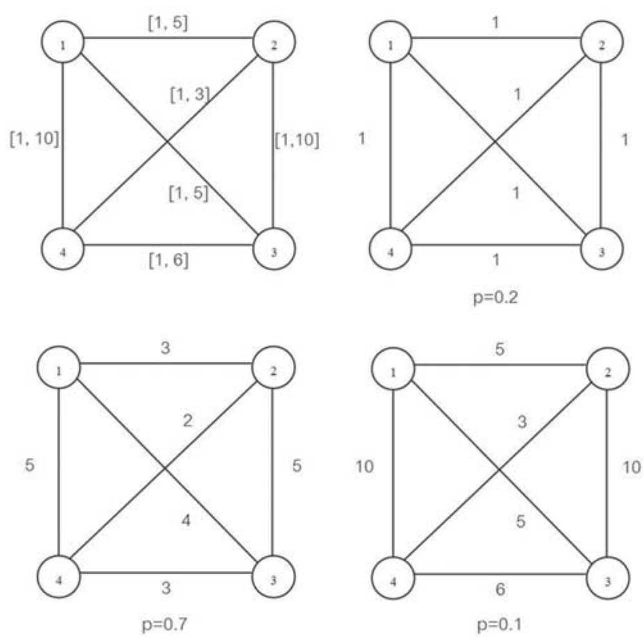

Figure 2 PSBA Approach

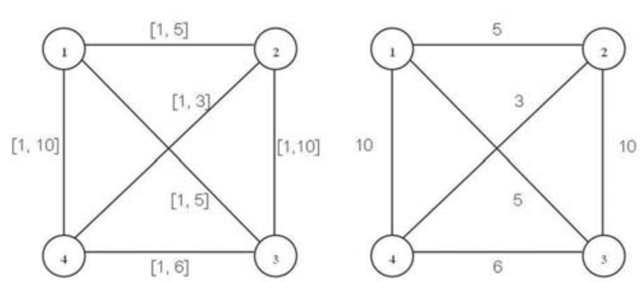

Figure 3 WCA Approach

The objective function in equation (1) is made explicit as equation (2). The x_s^* is an optimum solution and $t_{p,s}$ is a travel time which is realized for the scenario, s with probability, p. The constraints of this problem are the same as the original VRP for deterministic optimization. The mathematical formulation of the VRP defines x_{ijk} as a decision variable where $x_{ijk} = 1$ if there exists arc $\{i, j\}$ for vehicle number k is on the route and 0 else.

The memetic algorithm for this problem includes the modified push-forward insertion method (MPFIH) and λ local search descent (λ-LSD) with tabu search (TS). The MPFIH is a heuristic method for inserting customers into a route based on the push-forward insertion method of Solomon (1987) and Thangiah (1996). It is an efficient method for computing the inserting a new customer into the route.

The λ-LSD is a type of neighborhood search that the set of all neighbors is generated by the LSD for a given integer, λ. It is a sequential search which selects all possible combination of different pairs of routes. Then the TS is used as a diversification method that prevents the algorithm falling in a local optima. The TS is often referred to a "meta-heuristic". The TS is used to swap nodes or re-arrange a sequence of customers for each route. The TS is a memory-based search strategy which guides the local search descent method (LSD) to continue their search beyond local optimum (Glover, 1989, 1990). When a local optimum is encountered, a move to the best neighbor is made to explore the solution space, even though this may cause deterioration in the objective function value. The TS seeks the best available move that can be determined in a reasonable amount of time. If the set of neighborhood is large or their elements are expensive to evaluate, candidate list strategies are used to help restrict the number of solutions examined on a given iteration.

This memetic algorithm for the robust discrete optimization can be summarized as follows:

1. Construct uncertain travel time data by a base case scenario, s_b from uniform distributed, $t_{sb} \in [l_{ij}, u_{ij}]$

2. Generate for all possible scenarios by percentage of uncertainty, β and probability, p i.e. $t_{p,s} \in [(1-\beta)l_{ij}, (1+\beta)u_{ij}]$ where $0 \le p \le 1$

3. Selected appropriate criterion, Z_R

4. Construct the mathematical model formulation $Z = \min\{h_x(S) \,|\, g_s(X) \le h_x(S)\}$, where, $s \in S; X \in 4_{s\in S} F_s$ and $g_s(X) = \dfrac{f(X, t_{p,s}) - Z_s}{Z_s}$

5. Construct the initial solution x_0 by MPFIH method.

6. For each individual s, evaluate the minimum solution, $g_s(X) = \min\{F(x, t_{p,s}) : x \in X\}$ by using the λ-LSD and TS

7. For each individual x, evaluate the maximum solution among all possible scenarios s, $h_x(S) = \max\{F(x, t_{p,s}) : s \in S\}$

8. Find x_R that is the robust solution which $\min h_x(S)$ belongs to the objective function and constraints.

Sample Problem

To implement the memetic algorithm, the source code is created on MATLAB computing software. The algorithm is applied to solve the Solomon's VRP benchmarking problems, type R101 which has 25-customer nodes and a single depot. The vehicle capacity is 200 units. The perturbation parameter, β, controls percentage of uncertainty in travel times. When the traffic appears smoothly, β is thus low. Its value increases when the traffic jam, accident or the badly weather occurs. The minimum travel times $l_{ij} = (1-\beta) \text{ x } d_{ij}$, where d_{ij} is an Euclidian distance from i to j. The u_{ij} for each route is $l_{ij} = (1+\beta) \text{ x } d_{ij}$, where $0 < \beta \le 1$. Number of scenarios, $|\text{S}|$= 10 is considered in the experiments which the probabilities, p are equal to 0.05, 0.025, 0.1, 0.3, 0.1, 0.025, 0.085, 0.08, 0.2, 0.035, respectively.

Results and Discussion

For each experiment, the realizations of scenarios are computed to evaluate the quality of the routes, generated by the PSBA, SBA and WCA. The experiment observes how each type of route performs under random generated travel time situations. The performance measure, the ratio *R*, quantifies the relative extra travel times of the robust solution with respect to the optimum travel times for each scenario. It is calculated by $R = \dfrac{Z_r - Z_s}{Z_s}$ where Z_r is the optimal objective value of the robust counterpart, and Z_s is the optimal objective function value of each scenario belonging to the deterministic VRP. This ratio gives information on how much extra travel times will be incurred during the robust route to protect against all possible realizations of the traffic condition. When R is close to zero, it indicates a high performance solution. The performance is compared the solutions by the average relative ratio, $\overline{R}$, when $|\text{S}|$=10 then $\overline{R} = \dfrac{1}{10}\sum_{s=1}^{10}\dfrac{Z_r - Z_s}{Z_s}$. The corresponding results are shown in Table 1 and Figure 4. The results clearly show that the performance of the PSBA is better than or more competitive compared to the existing robustness approaches. Its performance is nearly the SBA while the WCA is overly conservative.

Conclusions and Future Work

The modeling of VRP with uncertain travel times aims to minimax robust discrete optimization by using the PSBA. The results are compared between the two approaches, SBA and WCA. The experiments show clearly that the robust solution from the PSBA is potential and strong for the perturbation of traffic conditions and the changes in uncertainty. The SBA is nearly performance and WCA is overly conservative approach. For future work, the extended research focuses on large scale problem. The developing algorithm with the PSBA or SBA is used to solve the large scale VRP, *i.e.* n = 50, 100, 200, 1000 to illustrate its performance when the number of customers increases.

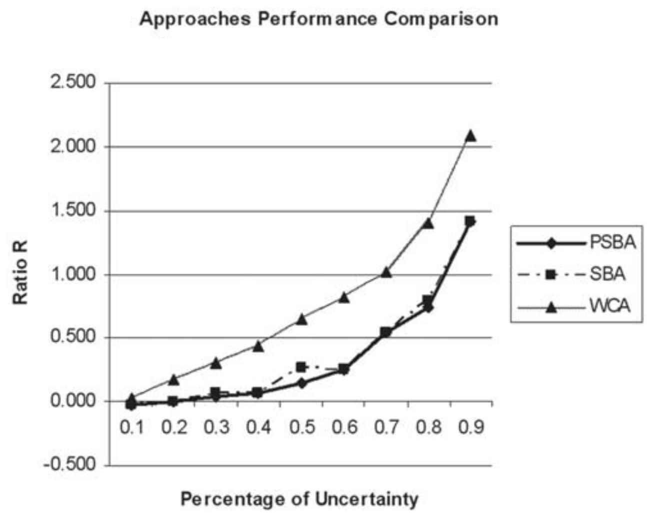

Figure 4 Approaches performance comparison

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Table 1 The result of the experiment

β	Avg.Optimal Value		PSBA		SBA		WCA	
	NV	TT	NV	TT	NV	TT	NV	TT
0.1	5.08	518.75	4	509.08	4	507.22	4	533.38
0.2	4.72	494.28	5	493.12	5	493.12	4	577.74
0.3	4.89	481.61	4	499.01	5	512.95	4	631.12
0.4	4.89	471.75	4	500.87	4	500.87	4	679.51
0.5	4.70	448.23	4	517.11	5	569.37	5	742.1
0.6	4.80	435.70	5	546.59	5	546.59	5	792.09
0.7	4.73	417.92	6	647.31	6	647.31	5	843.29
0.8	4.87	363.84	6	632.42	6	655.49	4	873.06
0.9	5.10	319.11	6	771.68	6	771.68	3	985.18

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